Calorons and BPS monopoles with non-trivial holonomy in the confinement phase of SU(2) gluodynamics*

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With the help of the cooling method applied to SU(2) lattice gauge theory at non-zero $T \leq T_c$ we present numerical evidence for the existence of superpositions of Kraan-van Baal caloron (or BPS monopole pair) solutions with non-trivial holonomy, which might constitute an important contribution to the semi-classical approximation of the partition function.

A large number of applications to hadronic phenomenology has proven the instanton liquid model to be a powerful non-perturbative calculational framework of QCD. Whereas fundamental non-perturbative features like spontaneous chiral symmetry breaking and the large η'-mass can be understood in terms of instantons, quark confinement seems to be related to other excitations like Abelian monopoles or center vortices. That instantons constitute important contributions to the Euclidean QCD vacuum transition amplitude has found support from lattice investigations. What possibilities exist to extend the present instanton model? On one hand, correlations among instantons have to be taken into account. On the other, the question arises, whether other extrema of the Euclidean action also have to be taken into consideration in the semi-classical ansatz. What rôle do play extended monopoles?

For $T \neq 0$ the semi-classical approach [1] so far was based on superpositions of time-periodic instantons, the so-called calorons [2]. These HS calorons have trivial holonomy, i.e. the Polyakov loop at spatial infinity takes values in the center of the gauge group. Recently, Kraan and van Baal (KvB) have constructed a more general class of periodic and stable solutions of the Euclidean Yang-Mills field equations exhibiting non-trivial holonomy [3]. The HS calorons turn out to be a limiting case of KvB solutions. The main feature of the $SU(N_c)$ KvB solutions, having topological charge $Q_t = \pm 1$, is that they can dissolve into N_c separate static lumps of non-integer topological charge, each of them constituting a BPS monopole. Taken as a background field for the Dirac fermion operator, they lead to zero-modes concentrated only at one of these BPS monopoles [4], depending on the boundary conditions used for the fermion field. These features can be used to identify KvB solutions in smoothed lattice fields.

In this contribution we are discussing some new results (see [5]) of continuing lattice investigations presented also at previous LATTICE conferences. For pure Yang-Mills theory at non-zero temperature we identify low-action semi-classical configurations which are the background fields for equilibrium gauge fields in the path integral represen-

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tation of the partition function. We start from Monte Carlo (MC) generated lattice gauge fields. By standard relaxation ('cooling') we minimize the action down to metastable plateaus shown to be approximate solutions of the lattice field equations. We restrict ourselves to the $SU(2)$ case and use the Wilson action. The lattice size throughout this paper is $16^3 \times 4$, i.e. the deconfinement transition occurs at $\beta \equiv 4/g^2 = \beta_c \simeq 2.29$. We shall concentrate on the confinement phase. In contrast to our previous investigations we employ standard periodic boundary conditions (p.b.c.) both for the MC simulation and for cooling. For the action as a function of cooling iterations we observe shoulders or plateaus close to multiple values of the one-instanton action value $S_{\rm inst} = 2\pi^2\beta$. We stop cooling on plateaus when the second derivative of the action changes its sign from positive to negative values. At these stages of cooling the gauge fields are investigated with respect to the action density, the naively discretized topological density $q_t(x) = -\frac{1}{2^9\pi^2}\sum_{\mu,\nu,\rho,\sigma=\pm 1}^{\pm 4} \epsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}\,[U_{x,\mu\nu}U_{x,\rho\sigma}]$, the Polyakov loop variable $L(\vec{x}) = \frac{1}{2}\mathrm{tr}\prod_{t=1}^{N_t} U_{\vec{x},t,4}$ and the eigenvalues λ and eigenmode densities $\psi\psi^\dagger(x)$ of the non-Hermitean Wilson-Dirac operator $D[U]$, with $\sum_y D[U]_{x,y}\ \psi(y) = \lambda\ \psi(x)$. We have applied the Arnoldi method to both cases of time-antiperiodic and time-periodic boundary conditions of the fermion fields. The statistics for our investigations were $O(200)$ per β-value.

First we investigate the plateau configurations at $S \simeq S_{\rm inst}$ and below. There we have seen three types of non-trivial objects. First of all, with the highest frequency of 60 to 70%, we have found approximate solutions which show all indications for (anti)selfdual KvB solutions consisting of one or two separate lumps of topological charge. Both cases are characterized by opposite-sign peaks of the Polyakov loop and non-trivial ('asymptotic') holonomy values. The Wilson-Dirac operator exhibits always one distinct real ('zero'-) mode. For time-antiperiodic b.c. the related eigenfunction is localized just at the peak of the Polyakov loop which corresponds to so-called Taubes winding (see [3]). Repeating the eigenmode investigation with periodic b.c. provides eigenfunctions localized at the complementary Polyakov loop peak in exact correspondence with the analytic results [4]. We have denoted these configurations as calorons (CAL), if they were consisting of one topological lump and were non-static in the time-direction, whereas we called them dyon-dyon pairs (DD) if a dissociation into two lumps of (non-integer) topological charges became visible and their fields were static in time. A typical DD configuration

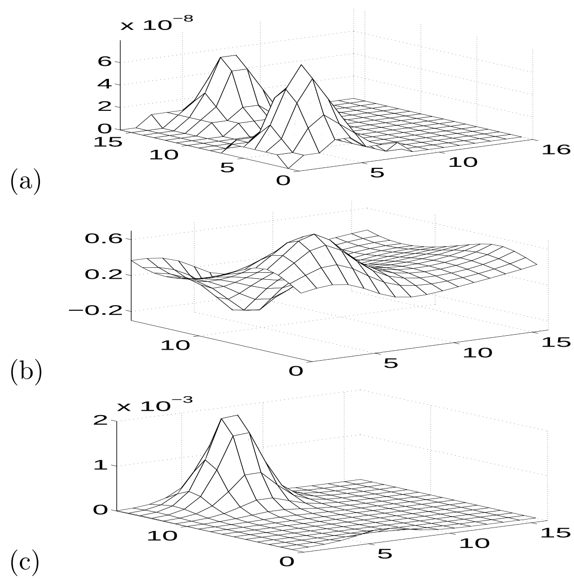


Figure 1. $2D$ cuts for a DD pair after cooling: (a) topological charge density, (b) Polyakov loop, (c) 'zero'-mode density for time-antiperiodic b.c.

has been drawn in Fig. 1. Fits of DD and CAL configurations with the analytic KvB solutions were done and have shown to work well. Therefore, we conclude that KvB solutions at lowest instanton action plateaus are found dominating for $T \leq T_c$.[5] The second class of very stable configurations (around 10 to 20%) were dyon-antidyon pairs ($D\overline{D}$) consisting of lumps of opposite half-integer topological charge and of equal-sign peaks of the Polyakov loop variable. The fermionic spectrum showed pairs of almost real eigenvalues

[5]The situation is completely different for $T > T_c$. Selfdual semi-classical fields become strongly suppressed under cooling if p.b.c. are applied.

complex conjugate to each other. These objects are not described by KvB solutions and look like peculiar superpositions of BPS monopoles of opposite topological charge. Their rôle is still unclear. Finally, in quite rare cases (less than 10%) we have seen pure magnetic objects like Dirac sheets or pure magnetic monopoles. In contrast, such objects seem to dominate the semi-classical structure at $T > T_c$ [6].

Higher action plateaus $S \simeq (2 \cdots 8)S_{\text{inst}}$ have been investigated for $T \leq T_c$ with the same methods and observables. One can easily identify superpositions of CAL's, D's and $\overline{D}$'s. The Polyakov loop variable as well as the fermionic modes with both kinds of boundary conditions allowed us to prove that these configurations always exhibit a pair structure (of Polyakov line peaks) supporting the view of superpositions of KvB solutions with an admixture of $D\overline{D}$-pairs. Of course, individual pairs are hardly identified.

What is the typical holonomy ascribed to these configurations? We have determined the distribution $P(L_\infty)$ of the Polyakov loop averaged over low action regions $M = \left\{ \vec{x} \;\mid\; \frac{1}{N_t}\sum_t s(\vec{x},t) \leq 10^{-4} \right\}$ in the form $L_\infty = \frac{1}{N_M}\sum_{\vec{x}\in M} L(\vec{x})$. The result is seen in Fig. 2. The histogram peaks around zero indicating the dynamical dominance of non-trivial holonomy. The

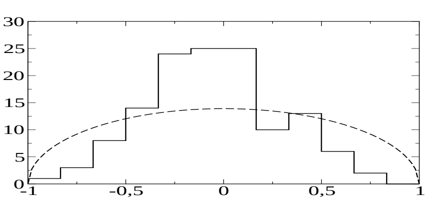
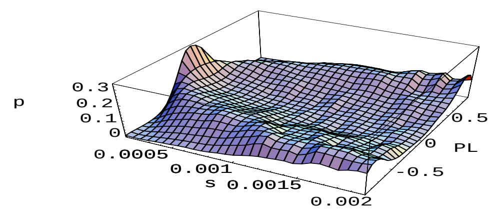


Figure 2. Histogram $P(L_\infty)$ for $S \simeq 4S_{\text{inst}}$ plateaus compared with the Haar measure (dashed line).

Polyakov line behaviour in different intervals of local action can be used in order to see that also complicated superpositions of D's and $\overline{D}$'s are similar to KvB-solutions. For low-L_∞ configurations we have found a local correlation between the Polyakov loop $L_(\vec{x})$ and the action density values $\varsigma(\vec{x}) = \frac{1}{N_t}\sum_t s(\vec{x},t)$ in the form of *conditional distributions* $P[L|\varsigma]$. In Fig. 3 we compare the result seen on four-instanton plateaus with that of lattice-discretized KvB solutions with randomly distributed parameters.

We compared also with randomly distributed 'old-fashioned' HS calorons of trivial holonomy. The latter provide a completely different picture. Therefore, the conditional distributions $P[L|\varsigma]$ are a hint in favor of KvB solutions, also for ensembles with several lumps of topological charge.



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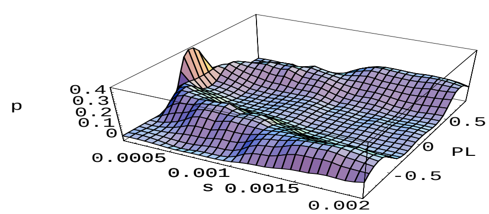

(b)

Figure 3. Conditional distributions $P[L|\varsigma]$ (a) at plateaus $S \simeq 4S_{\text{inst}}$, (b) for randomly distributed KvB solutions.

We would like to conclude that an improvement of the semi-classical caloron approach to the path integral at $0 < T < T_c$ should take into account superpositions of solutions with non-trivial holonomy.

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